

On the Presentation of Miller's Theorem

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Abstract—Miller's theorem is an important analysis tool. Its presentation in many introductory electronic circuits texts often leads to student misunderstandings with regard to its applicability. An alternative presentation that may improve students' understanding is suggested.

FOR years Miller's theorem has been presented in many introductory electronics circuits texts as shown in Fig. 1(a) and (b) [1]–[4]. Miller's original paper relates to the *Miller effect* and discusses only the input impedance; there is no mention of a *theorem*. The generalization to a theorem appears to be due to Millman and Halkias [1].

Millman points out in his discussion of Miller's theorem: "... this theorem will be useful in making calculations only if it is possible to evaluate K [A in Fig. 1(b)] by some independent means" [2]. Unfortunately, many students remember the picture of Fig. 1(b) without the limitation with regard to its applicability. In particular, they come to believe that the transformed circuit simplifies the calculation of the output impedance as well as the input impedance! This is not the case; since to make Y_{out} be Y_2 one must evaluate a new A when the circuit is excited from its output terminals. This is required because a voltage transfer ratio is not a reciprocal quantity. The A in Fig. 1(b) is a *forward* voltage ratio, and consequently that circuit can be used to calculate forward gain and input impedance only. If Y_2 is to be used in an output impedance calculation the *reverse* voltage ratio must be used for A .

To avoid the students' confusion relative to the applicability of the Miller equivalent circuit of Fig. 1(b), the writer would like to propose the circuit shown in Fig. 1(c). As in Fig. 1(b), the bridging admittance Y is replaced at the input by the shunt admittance

$$Y_1 = Y(1 - A).$$

To evaluate this one must know (or calculate) the forward voltage ratio

$$A = \frac{V_2}{V_1}.$$

It follows that once V_1 has been calculated, including the effect of Y_1 , V_2 is completely determined. This is as indicated by the voltage controlled voltage source in Fig. 1(c). It has been the writer's experience that the circuit of Fig. 1(c) helps to remind students that the Miller equivalence is useful primarily for input and forward transmission calculations.

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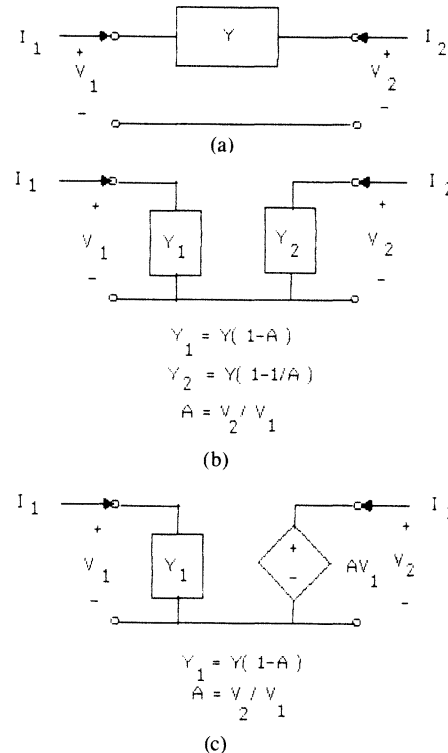


Fig. 1. Circuit representations of Miller's theorem. (a) A typical circuit. (b) The Miller equivalence. (c) Suggested improved circuit representation.

Example

A circuit which includes the essentials of a common-emitter *BJT* or a common-source *FET* is shown in Fig. 2. For this circuit it is easily shown that

$$\frac{V_0}{v} = \frac{-g_m + sC_{12}}{1/R_L + sC_{12}} \quad (1)$$

and, therefore, the Y_1 and Y_2 of Fig. 1(b) are

$$Y_1 = sC_{12} \left(\frac{1 + g_m R_L}{1 + sR_L C_{12}} \right) \quad (2)$$

and

$$Y_2 = sC_{12} \left(\frac{1 + g_m R_L}{g_m R_L - sR_L C_{12}} \right). \quad (3)$$

The two Miller equivalent circuits corresponding to Fig. 1(b) and (c) are given in Fig. 3(a) and (b). The admittance Y_1 is that of a series *RC* circuit as indicated, but Y_2 is not a passive admittance. Nevertheless, either circuit can be used to calculate the forward gains V_o/V_s or v/V_s as well as the input admittance I_s/V_s . Neither circuit gives the output admittance nor the reverse transmission of Fig. 2

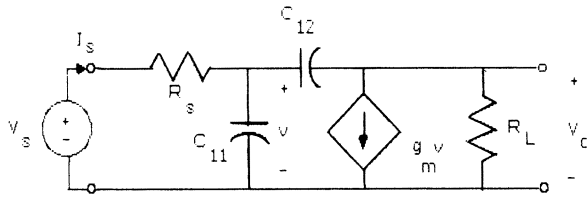


Fig. 2. Circuit for illustration of Miller's theorem.

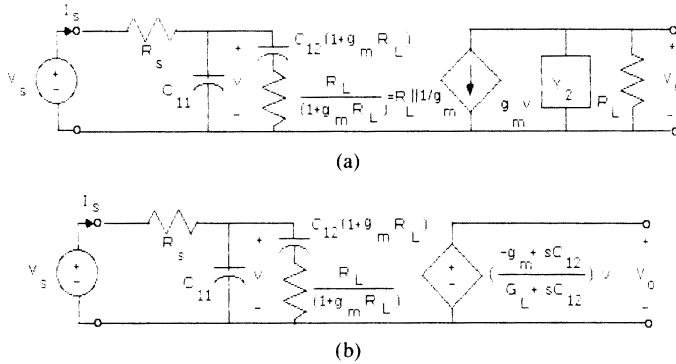


Fig. 3. Two "Miller" equivalent circuits.

directly. Unfortunately, many students (and several authors) look at Fig. 3(a) and conclude that the output admittance is

$$Y_{out} = 1/R_L + Y_2. \quad (4)$$

Using (2), one can easily show for this example circuit

$$Y_{out} = (1/R_L + sC_{12}) \left(\frac{g_m}{g_m - sC_{12}} \right) \quad (4')$$

which is *not* the output admittance of Fig. 2. The correct output admittance can be determined from either Fig. 3(a) or (b) by using

$$Y_{out} = \lim_{R_L \rightarrow 0} \left\{ \frac{V_o}{R_L} \right\} = \frac{I_{sc}}{V_{oc}} \quad (5)$$

or by a direct analysis of Fig. 2. In either case, one finds

$$Y_{out} = \frac{1}{R_L} + sC_{12} \left(\frac{1 + g_m R_s + sR_s C_{11}}{1 + sR_s(C_{11} + C_{12})} \right) \quad (6)$$

which differs significantly from (4') unless $R_s = 0$.

An examination of the many introductory electronic circuits texts available today, and in the recent past, shows that authors as well as students have been misled by the presentation of Miller's theorem summarized by Fig. 1(a) and (b). The author urges that future authors use the combination of Fig. 1(a) and (c) in their presentations plus a clear warning that the equivalence does not apply for the prediction of reverse transmission or output immittance. Gray and Searle give an excellent example of such a warning [5].

REFERENCES

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